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1970 J. Phys. A: Gen. Phys. 3 524

(http://iopscience.iop.org/0022-3689/3/5/009)

J. Phys. A: Gen. Phys., 1970, Vol. 3. Printed in Great Britain

On the Krutov model for deformed even-even nuclei

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MS. received 16*th March* 1970

Abstract. Rotational energy levels of the ground state band of a number of deformed even–even nuclei have been calculated using the models proposed by Krutov in 1968 and by Davydov and Filippov in 1958. The parameters, occurring in these models, have been evaluated by two different methods. The results are compared with the experimental values and the relative merits of the models discussed.

1. Introduction

Recently Krutov (1968 a, b) has suggested a new approach to the description of the rotation of deformed nuclei. His approach is based on a definition of collective motion in the nucleus as a change of the density distribution of nuclear matter in time.

Krutov (1968 a, b) and Krutov and Zackrevsky (1969 a) have applied this approach to nuclei having a non-axial equilibrium shape. Krutov limits himself to the case when the coupling between the rotation and intrinsic motion can be neglected, i.e. an asymmetric rotator. The rotational Hamiltonian of the non-axial nucleus is then equal to

$$H_{\mathrm{rot}} = \frac{\hbar^2}{2}\sum_{\nu=1}^{3}\frac{I_\nu{}^2}{F_\nu} \tag{1}$$

where I_ν is the angular momentum projection on the ν axis of the nucleus-fixed system; F_ν is the effective moment of inertia under the rotation around the ν axis.

According to the model suggested by Krutov,

$$F_\nu = \int \tilde{\rho}_\nu\{r^2-(x_\nu{}')^2\}\,\mathrm{d}\boldsymbol{r}' \tag{2a}$$

$$\tilde{\rho}_\nu(\boldsymbol{r}') = \rho(\boldsymbol{r}')-\{\rho_{\mathrm{min}}(\boldsymbol{r}')\}_\nu \tag{2b}$$

where $\rho(\boldsymbol{r}')$ is the nuclear mass density distribution; $\{\rho_{\mathrm{min}}(\boldsymbol{r}')\}_\nu$ is the minimum density at the point r' under the rotation of the nucleus around the ν axis. $\boldsymbol{r}'$ or $x_{\nu=1,2,3}$ represent the coordinates in the nucleus-fixed system.

Assuming the uniform distribution of the density $\rho(\boldsymbol{r}')$ and using equation (2), Krutov obtains

$$F_{1,2} = F_{\mathrm{rs}}\frac{3}{2}\left(\frac{5}{\pi}\right)^{1/2}\beta\left\{1-\frac{31}{112}\left(\frac{5}{\pi}\right)^{1/2}\beta+\frac{1\cdot43}{\pi}\beta^2\pm\frac{1}{\sqrt{3}}\gamma-\ldots\right\} \tag{3a}$$

$$F_3 = F_{\mathrm{rs}}\left(\frac{15}{\pi}\right)^{1/2}\beta\gamma\left\{1-\frac{4}{7}\left(\frac{5}{\pi}\right)^{1/2}\beta+\ldots\right\} \tag{3b}$$

where β is the total deformation parameter of the nucleus, γ is the non-axiality parameter and F_{rs} is the moment of inertia of a rigid sphere possessing the nuclear mass and radius ($F_{\mathrm{rs}} = \frac{2}{5}M_AR^2$, $R = 1\cdot216\ A^{1/3}$ fm).

Like the Davydov–Filippov (1958) model, this model also gives rise to a ground state band with the spin sequence $J\pi = 0+, 2+, 4+, 6+$ etc., and a gamma band with the spin sequence $2+, 3+, 4+, 5+$ etc. For the Hamiltonian (1) it is possible to obtain analytical expressions for $E_1(2+)$ and $E_\gamma(2+)$ in terms of F_1, F_2 and F_3. $E_1(2+)$ is the energy of the $2+$ level in the ground state band and $E_\gamma(2+)$ that of the $2+$ level in the gamma band. Krutov and Zackrevsky (1969 a) used the experimental values of the two $2+$ states to calculate β and γ for 23 even–even nuclei. They restricted themselves to such even–even nuclei for which the following relation is closely satisfied:

$$E_1(2+)+E_\gamma(2+) = E_\gamma(3+). \tag{4}$$

For an asymmetric rotator, equation (4) is exactly satisfied (Davidson 1968). The degree to which this relation is satisfied can be used as a criterion for ignoring the coupling between the rotation and intrinsic motion.

Krutov and Zackrevsky (1969 a, b) have also calculated the reduced probabilities for $E2$ transitions between two rotational levels of the non-axial nucleus, the magnetic moments of the lowest rotational states and the probabilities of magnetic dipole transitions between these states. These results are expressed in terms of β_e (the parameter of the total deformation of charge) and γ_e (the charge non-axiality parameter). Because of the difference between the mass and charge distributions, these two parameters are different from β and γ. The calculated values of transition probabilities and gyromagnetic factors are in reasonable accord with the experimental values, lending support to the applicability of Krutov's model to such nuclei. These results show the consistency of the assumptions involved in the parameters β_e and γ_e, but do not say anything about β and γ.

A satisfactory nuclear model should also be able to reproduce the observed energy levels. In the present paper we have obtained the energies of the ground state band levels for Krutov's model. There are certain similarities between the Davydov–Filippov (DF) model and the Krutov model. Both are asymmetric rotator models, both disregard the vibration–rotation interaction and both have two parameters as far as the energy levels are concerned. A comparison of the two would be appropriate. The Hamiltonian operator in the case of the DF model is given by

$$H = \sum_{i=1}^{3} \frac{AI_i^2}{2\sin^2\{\gamma_{\mathrm{DF}}-(2\pi/3)i\}} \tag{5}$$

where A and γ_{DF} are parameters and I_i are operators of the projections of the total angular momentum along the body-fixed axes of the nucleus. We have obtained the ground band energies for the DF model also and the calculated values from the two models have been compared with the experimental values.

2. Results

In the case of the asymmetric rotator two-parameter models, the values of the parameters may be obtained by either of the following two methods.

(i) From $E_1(2+)$ and $E_\gamma(2+)$. For Krutov's model, Krutov and Zackrevsky (1969 a) have calculated the parameters β and γ by this method and these are reproduced in table 1. To compare the results obtained from these parameters with those of the DF model, the parameter γ_{DF} occurring in the DF model was also calculated from $E_1(2+)$ and $E_\gamma(2+)$ and the resulting values are shown in table 2.

Table 1. Parameters of Krutov's model as calculated by methods (i) and (ii)

Nucleus	Method (i) β	Method (i) γ (deg)	Method (ii) β	Method (ii) γ (deg)
^{150}Nd	0·239	3·567	0·2210	29·08
^{152}Sm	0·221	3·167	0·2351	29·04
^{154}Sm	0·331	1·783	0·3402	17·20
^{154}Gd	0·218	4·000	0·2273	28·61
^{156}Gd	0·298	2·450	0·3069	18·14
^{158}Gd	0·327	2·117	0·3336	14·46
^{160}Gd	0·338	2·333	0·3459	15·30
^{160}Dy	0·293	2·867	0·2997	16·02
^{162}Dy	0·308	2·884	0·3147	13·72
^{164}Dy	0·349	2·884	0·3387	13·23
^{164}Er	0·229	4·567	0·2732	15·46
^{166}Er	0·286	3·383	0·3025	14·56
^{168}Er	0·293	3·100	0·2975	11·93
^{170}Er	0·289	2·717	0·2950	12·56
^{172}Yb	0·285	1·683	0·2899	11·99
^{176}Yb	0·263	2·050	0·2682	14·20
^{182}W	0·202	2·633	0·2059	13·78
^{186}W	0·158	5·583	0·1618	16·39
^{188}Os	0·123	8·467	0·1262	25·09
^{190}Os	0·100	12·0		
^{192}Os	0·089	14·834		
^{194}Pt	0·055	21·0		
^{232}Th	0·272	2·017	0·2804	15·65

Table 2. Parameters of the Davydov–Filippov model as calculated by methods (i) and (ii)

Nucleus	Method (i) γ_{DF} (deg)	Method (ii) γ_{DF} (deg)
^{150}Nd	13·91	22·15
^{152}Sm	13·23	22·13
^{154}Sm	9·54	15·87
^{154}Gd	13·84	22·00
^{156}Gd	11·05	16·62
^{158}Gd	10·33	14·07
^{160}Gd	10·87	14·60
^{160}Dy	11·90	15·22
^{162}Dy	11·94	13·53
^{164}Dy	12·30	13·11
^{164}Er	12·88	14·93
^{166}Er	12·67	14·18
^{168}Er	12·35	12·29
^{170}Er	11·58	12·76
^{172}Yb	9·26	12·37
^{176}Yb	10·15	14·04
^{182}W	11·38	13·89
^{186}W	16·03	15·84
^{188}Os	19·16	20·75
^{190}Os	22·28	23·39
^{192}Os	25·19	25·33
^{194}Pt		
^{232}Th	10·03	15·03

No value is shown for ^{194}Pt as for this nucleus $E_\gamma(2+)/E_1(2+)$ is less than two. The experimental data employed in the calculations are shown in table 3. Energy levels for the ground state band were calculated from the Hamiltonian (5) for the DF model and from the Hamiltonian (1) for the Krutov model and the results are compared with the experimental values in figures 1(a) and 1(b).

Table 3. Experimental data employed in the determination of parameters

Nucleus	$E_1(2+)$ (keV)	$E_1(4+)$ (keV)	$E_\gamma(2+)$ (keV)	y	$100\,\rho$
^{150}Nd	132	397	1060	0·326	
^{152}Sm	121·78 ±0·05	366·4 ±0·3	1087	0·325	−2·17
^{154}Sm	81·99 ±0·05	267 ±1	1440	0·077	−0·13
^{154}Gd	123·07 ±0·05	371·2 ±0·2	999	0·317	−0·80
^{156}Gd	88·967 ±0·005	288·16 ±0·05	1154	0·094	−0·65
^{158}Gd	79·51 ±0·01	261·45 ±0·05	1185	0·045	−0·28
^{160}Gd	75·3 ±0·5	247 ±2	1010	0·053	
^{160}Dy	86·8	283·8	966·1	0·064	0·35
^{162}Dy	80·6	265·6	890	0·038	
^{164}Dy	73·39 ±0·05	242·2 ±0·1	761·8	0·033	0·84
^{164}Er	91 ±1	298 ±3	858	0·059	0·32
^{166}Er	80·6 ±0·05	264·9 ±0·2	787	0·047	0·88
^{168}Er	79·8 ±0·5	264 ±0·5	822	0·025	0·53
^{170}Er	79 ±5	261 ±2	930	0·055	
^{172}Yb	78·7 ±0·5	260·3 ±1	1468	0·026	−0·21
^{176}Yb	82·1 ±0·5	270 ±3	1270	0·045	
^{182}W	100·1 ±0·05	329·4 ±0·05	1222	0·043	−0·67
^{186}W	122·5	399	730	0·076	
^{188}Os	155 ±0·1	477·9 ±0·1	633	0·250	−0·23
^{190}Os	186·7 ±0·1	547·8 ±0·1	557·9	0·399	−1·53
^{192}Os	205·79	580·4	489·1	0·513	0·65
^{194}Pt	328·5 ±1	811·1 ±2	622·1	0·864	
^{232}Th	49·8 ±0·1	163 ±1	788	0·060	

Most of the data are from the compilations of Lederer *et al.* 1967 and Mariscotti *et al.* 1968. Ground state band values for ^{160}Dy and ^{162}Dy are from Ewan and Andersson 1968. For purposes of identification, $E_\gamma(2+)$ was taken to be the energy of that 2+ state which most closely satisfies relation (4).

(ii) From $E_1(2+)$ and $E_1(4+)$. It is known that for the DF model this method gives better results for the ground state bands of a number of nuclei than those obtained by the method (i) (De Mille *et al.* 1959, Moore and White 1960, Varshni and Bose 1970). The parameter γ_{DF} in the DF model was determined from the ratio $R(4) = E_1(4+)/E_1(2+)$ and the values obtained are shown in table 2. No value is shown against ^{194}Pt as for this nucleus $R(4) < 8/3$ (the limiting value for the DF model). The parameters β and γ for the Krutov model were also determined from $E_1(2+)$ and $E_1(4+)$ by an iterative process with the constraints $0 < \beta < 1$ and $0 < \gamma < 30°$. The calculated values thus obtained are shown in table 1. For ^{190}Os, ^{192}Os and ^{194}Pt, values of β and γ could not be obtained within the stipulated ranges.

Using these parameters, energy levels for the ground-state band were calculated for the two models and the results are shown, along with the experimental values, in figures 2(a) and 2(b).

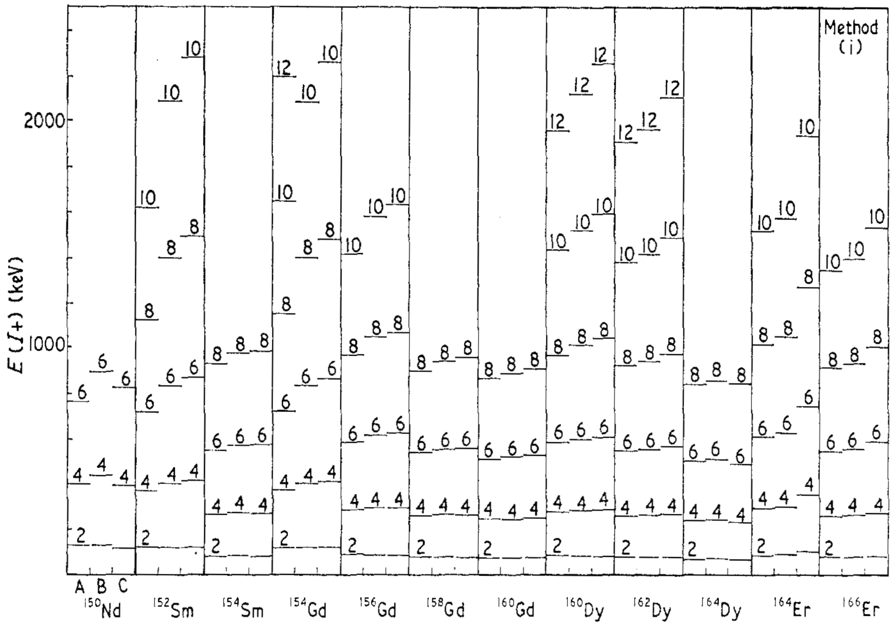

Figure 1(*a*). Experimental and calculated energy levels. The parameters were determined from $E_1(2+)$ and $E_\gamma(2+)$. There are three columns for each nucleus, see e.g. ^{150}Nd—the first one A shows the experimental energy levels, the second one B, those calculated from the Davydov–Filippov model and the third C those calculated from the Krutov model. Spins have been identified; parity in all cases is positive.

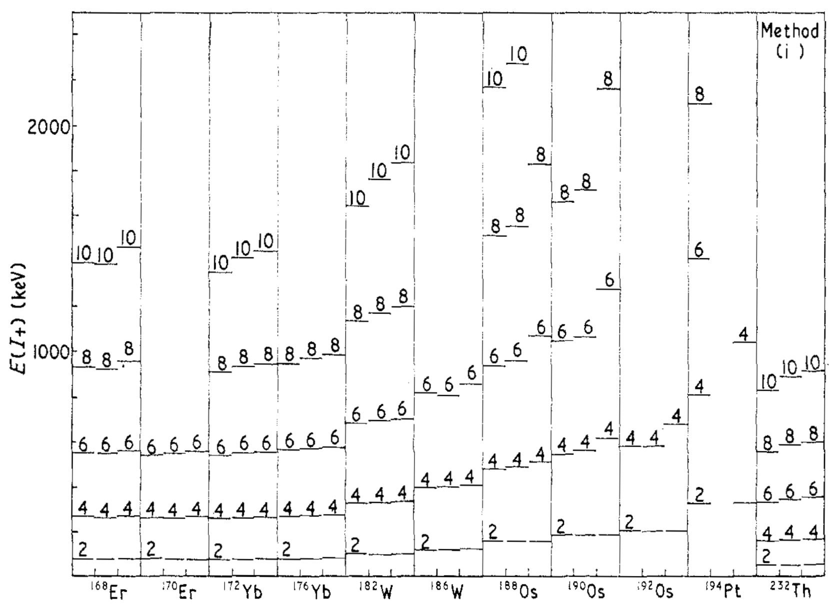

Figure 1(*b*). See explanation below figure 1(*a*).

3. Discussion

3.1. *Results by method* (*i*)

Generally speaking, results by both the models are poor, Davydov–Filippov ones being the better of the two. We may note here that the calculated values of 2+ levels of ^{150}Nd and ^{164}Er by the Krutov model do not coincide with the observed ones; presumably Krutov and Zackrevsky (1969 a) used experimental values for these two energy levels different from those given in table 3.

The pattern of results obtained by the two models is similar; if for a given nucleus the DF model gives, relatively speaking, good results, so does the Krutov model. Similarly if the DF model gives very poor results for some nucleus, so does the Krutov one. This can be attributed to the fact that both are pure rotator models and the degree of agreement would depend on how closely a given nucleus corresponds to this picture.

For a rigid asymmetric rotator, the following ratio (Davidson 1968) should be equal to zero:

$$\rho = \frac{E_1(2+)+E_\gamma(2+)-E_\gamma(3+)}{E_1(2+)+E_\gamma(2+)}. \tag{6}$$

The degree of deviation of this ratio from zero can give an idea of the influence of the rotation–vibration interaction. In column 6 of table 3 we have recorded the values of 100 ρ for such nuclei for which data are available for the three energy levels. By definition, we have considered here only such nuclei for which equation (4) is closely satisfied, consequently even small uncertainties in the individual energy levels lead to a very large percentage uncertainty (sometimes of the order of 100%) in the value of ρ. This makes a correlation of this ratio with the results of figures 1(*a*) and (*b*) somewhat difficult and inconclusive.

We can, however, turn to another criterion, which is based on the Bohr-Mottelson model. For a rigid axially symmetric rotator the following quantity

$$y = \frac{10}{3} - \frac{E_1(4+)}{E_1(2+)} \tag{7}$$

may be considered to be an approximate measure of the strength of the rotation–vibration interaction. This quantity is tabulated in column 5 of table 3.

A perusal of table 3 and figures 1(*a*) and 1(*b*) shows that the degree of agreement between theory and experiment is closely correlated with the value of y. As an illustration we may quote here three cases:

^{164}Dy	agreement good,	$y = 0{\cdot}033$
^{152}Sm	agreement poor,	$y = 0{\cdot}325$
^{194}Pt	no agreement,	$y = 0{\cdot}864$.

Thus it appears highly likely that the differences between the observed and the calculated values are due to the neglect of the β-vibration in the two models. For the Davydov–Filippov case, an improved model which incorporates the β-vibration does exist—the Davydov–Chaban (1960) model. For the Krutov model, to a first degree of approximation, the effect of β-vibration can be allowed for by expressing the energy of a level as

$$E(I) = E_{\mathrm{K}}(I) - b\{E_{\mathrm{K}}(I)\}^2 \tag{8}$$

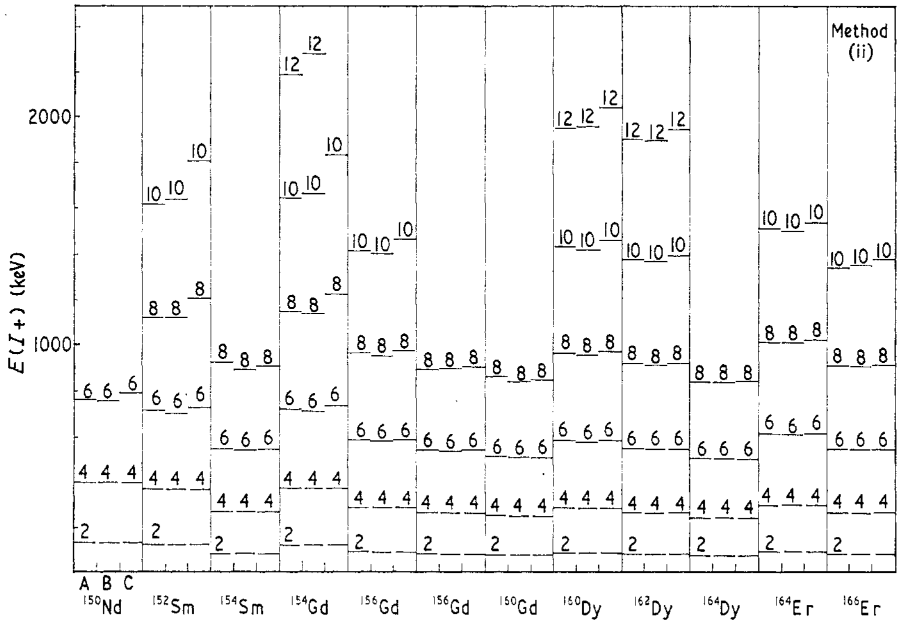

Figure. 2(a). Experimental and calculated energy levels. The parameters were determined from $E_1(2+)$ and $E_1(4+)$. There are three columns for each nucleus, see e.g. ^{150}Nd—the first one A shows the experimental energy levels, the second one B those calculated from the Davydov–Filippov model and the third C those calculated from the Krutov model. Spins have been identified; parity in all cases is positive.

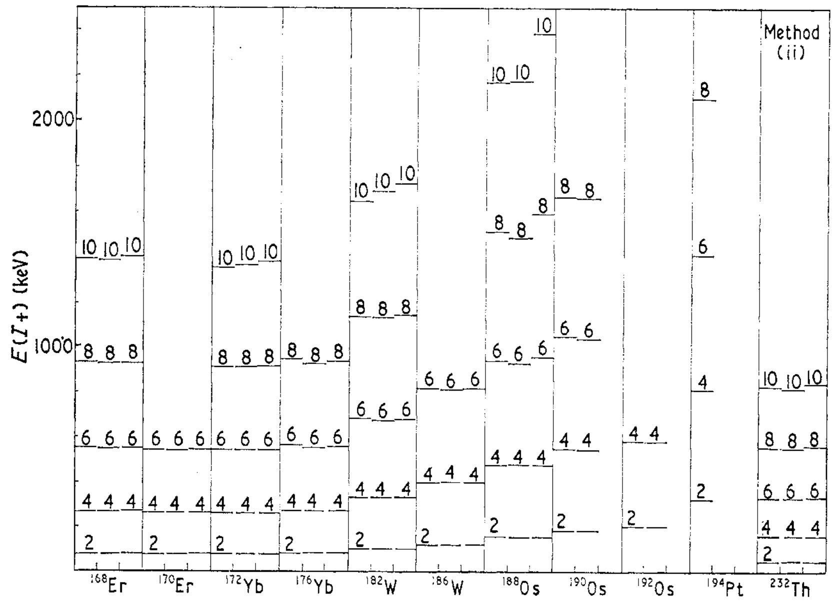

Figure 2(b). See explanation below figure 2(a).

where $E_K(I)$ represents the energy of the level given by the Krutov model in the rigid case, and b is a parameter.

3.2. *Results by method (ii)*

At first sight it may appear that this method would almost certainly lead to an improvement in the calculated values. This, however, is not necessarily so in all

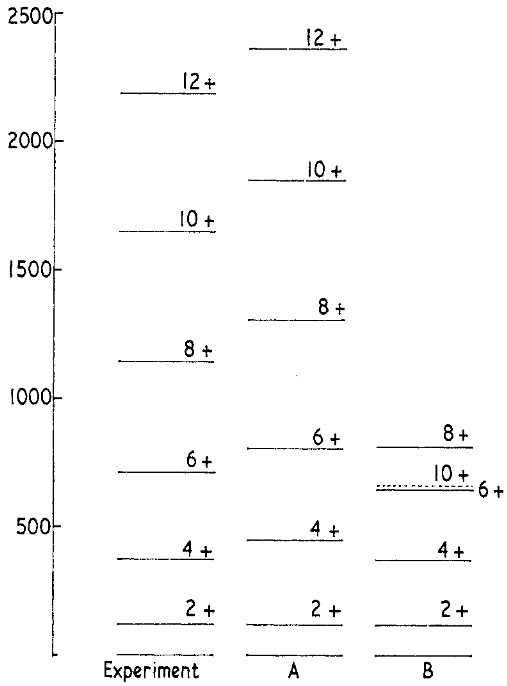

Figure 3. Energy levels of ^{154}Gd. A, Parameters of the Bohr–Mottelson model determined from $E_1(2+)$ and $E_\gamma(2+)$. B, Parameters determined from $E_1(2+)$ and $E_1(4+)$.

cases. As an illustration, in figure 3 we show the results obtained from a two-parameter Bohr–Mottelson equation:

$$E = AI(I+1)+BI^2(I+1)^2. \tag{9}$$

In the case indicated in figure 3A, A and B were obtained from $E_1(2+)$ and $E_\gamma(2+)$ (contribution of E_β to B was not taken into account). In the second case, figure 3B, A and B were determined from $E_1(2+)$ and $E_1(4+)$. It would be noticed that in B the agreement has worsened.

The results shown in figures 2(*a*) and 2(*b*) for Krutov and DF models show a very marked improvement over those obtained by method (i). For 6+ and 8+ levels, the results by both the models are of comparable agreement with the experimental data. However, for higher spin levels, i.e. 10+ and 12+, the Davydov–Filippov model gives better results than the Krutov one.

A comparison of the results by the two methods for the DF model shows an interesting feature. In method (i) we find that, for a given nucleus, the differences between the observed and the calculated energy levels are in the same direction. In method (ii), by determining the parameters from $E_1(2+)$ and $E_1(4+)$ one is sort of partially correcting for the rotation–vibration interaction. In figures 2(*a*) and 2(*b*) it should be noticed that in several cases the sign of (calculated − observed) tends to change as one goes to higher spin levels; ^{152}Sm and ^{154}Gd are good examples.

A comparison of γ_e and γ (method (i)) as determined by Krutov and Zackrevsky (1969 a) indicates that the charge distribution is more asymmetric than the mass distribution in most of the nuclei considered here. However, if we compare γ_e and γ (method (ii)), the reverse appears to be the case. We must add here, though, that the latter comparison is not very meaningful because γ_e was obtained by a method which would correspond to method (i). It would, of course, be better to compare γ(ii) with γ_e obtained from the ground state intraband transition probabilities; however, no such calculations are available.

For the Krutov model (table 1), it would be noticed that the values of β obtained by the two methods are very close, but the γ values are very different, being rather large and somewhat unrealistic in the second case.

The differences between the γ_{DF} as obtained by the two methods are usually much less than those for the γ of the Krutov model. On intuitive grounds, the behaviour of γ_{DF} appears to be closer to the physical situation; the rotation–vibration interaction is after all not very large and it should not drastically change the asymmetry of the nucleus.

Acknowledgments

The authors thank the National Research Council of Canada for financial support.

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